Gr. 12 AP Maths Prelims 2014

MEMO

Section A

Question 1

TP:

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Let $n = 1$ ✓

LHS: $1^2 = 1$

RHS: $\frac{1(1+1)(2(1)+1)}{6}$

$= \frac{1 \cdot 2 \cdot 3}{6} = \frac{6}{6} = 1$

✓

LHS = RHS

Assume true for $n = k$ ✓

$$1^2 + 2^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$$ ✓

Let $n = k+1$

$$1^2 + 2^2 + \ldots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$ ✓ ✓

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$ ✓

$$= \frac{(k+1)(k(2k+1) + 6(k+1))}{6}$$ ✓

$$= \frac{(k+1)(2k^2 + k + 6k + 6)}{6}$$ ✓

$$= \frac{(k+1)(2k+3)(k+2)}{6}$$ ✓

$$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$ ✓ ✓

$\therefore$ By principle of MI, statement is true $\forall n \in \mathbb{N}$

[12]

Question 2

2.1 2.1.1 $4 - 3e^{x+6} = 0$

$3e^{x+6} = 4$ ✓

$e^{x+6} = \frac{4}{3}$ ✓

$\ln \frac{4}{3}$ ✓ $= x + 6$

$\ln \frac{4}{3} - 6$ ✓ $= x$

$-5{,}71 = x$

4

2.1.2 $\ln(\ln 3x) = 0$

e^0 ✓ $= \ln 3x$

1 ✓ $= \ln 3x$

e^1 ✓ $= 3x$

$\frac{e}{3}$ ✓ $= x$

$0{,}91 = x$

4

2.1.3 $(e^x \cdot e^2)^3 = e^{-9}$

$e^{3x} \cdot e^6$ ✓ $= e^{-9}$

e^{3x+6} ✓ $= e^{-9}$ ✓

$3x + 6 = -9$

$3x = -15$

$x = -5$ ✓

4

2.1.4 $(\ln x)^3 = \ln x^4$

$(\ln x)^3 - 4\ln x = 0$ ✓

$\ln x((\ln x)^2 - 4) = 0$

$\ln x = 0$ or $\ln x - 2 = 0$

$e^0 = x$ $\quad$ e^2 ✓ $= x$

$x = 1$ ✓ $\quad$ $7{,}39 = x$

or $\ln x + 2 = 0$

$e^{-2} = x$

$0{,}14 = x$ ✓

4

2.1.5 $e^{|x-3|} = 2$

e^{x-3} ✓ $= 2$ if $x \geq 3$

$\ln 2 + 3 = x$

$3{,}69$ ✓ $= x$

e^{-x+3} ✓ $= 2$ if $x < 3$

$\ln 2 - 3 = -x$

$2{,}31$ ✓ $= x$

5

2.2 $\int_a^b \log_3 x \, dx = 5$

$\int_a^b \log_3 9x \, dx$

$= \int_a^b (\log_3 9 + \log_3 x) \, dx$ ✓ ✓

$= \int_a^b \log_3 9 + \int_a^b \log_3 x \, dx$ ✓

$= \int_a^b 2 \, dx + 5$ ✓ ✓

$= 2x]_a^b + 5 = 2b - 2a + 5$ ✓

7

[28]

Question 3

3.1 $x = 3 \pm 2i$

$(x-3)^2 = -4$

$x^2 - 6x + 9 + 4 = 0$

$x^2 - 6x + 13 = 0$

$2x^3 - 13x^2 + 32x - 13$

$(x^2 - 6x + 13)(2x - 1)$

$\therefore x = 3 + 2i$

$x = 3 - 2i$

$x = \frac{1}{2}$

8

3.2 $i|2-i| = p + qi$

$i(2-i) = 2i - i^2$

$= 2i + 1$

$\therefore p = 1 \quad q = 2$

$i(-2+i) = -2i + i^2$

$= -2i - 1$

$p = -1 \quad q = -2$

[5]

Question 4

4.1 $\lim_{x \to -2^-} ax + b = \lim_{x \to -2} ax^2 - 7 = \lim_{x \to -2^+} bx - 9$

$-2a + b = 4a - 7 = -2b - 9$

$-2a + b = 4a - 7$

$b = 6a - 7$

$4a - 7 = -2b - 9$

$4a - 7 = -2(6a - 7) - 9$

$4a - 7 = -12a + 14 - 9$

$16a = 12$

$a = \frac{12}{16} = \frac{3}{4}$

$b = -\frac{5}{2}$

8

4.2 $\lim_{x \to -2^-} f'(x) = \lim_{x \to -2^-} \frac{3}{4} = \frac{3}{4}$ ✓ ✓

$\lim_{x \to -2^+} f'(x) = \lim_{x \to -2^+} -\frac{5}{2} = -\frac{5}{2}$ ✓ ✓

4

not differentiable ✓

$\lim_{x \to -2^-} f'(x) \neq \lim_{x \to -2^+} f'(x)$

[12]

Question 5

On answer sheet

Question 6

6.1 $\frac{x^2 + x - 4}{(x^2+3)(x+1)} = \frac{Ax+B}{(x^2+3)} + \frac{C}{x+1}$ ✓ ✓ cover-up $C = -1$ ✓

$x^2 + x - 4 = (Ax+B)(x+1) - 1(x^2+3)$ ✓ ✓

$= Ax^2 + Bx + Ax + B - x^2 - 3$

$= x^2(A-1) + x(A+B) + B - 3$ ✓ ✓

$A - 1 = 1$

$A + B = 1$

$B - 3 = -4$

$B = -1$ ✓

$A = 2$ ✓

10

$\therefore \frac{2x-1}{x^2+3} - \frac{1}{x+1}$

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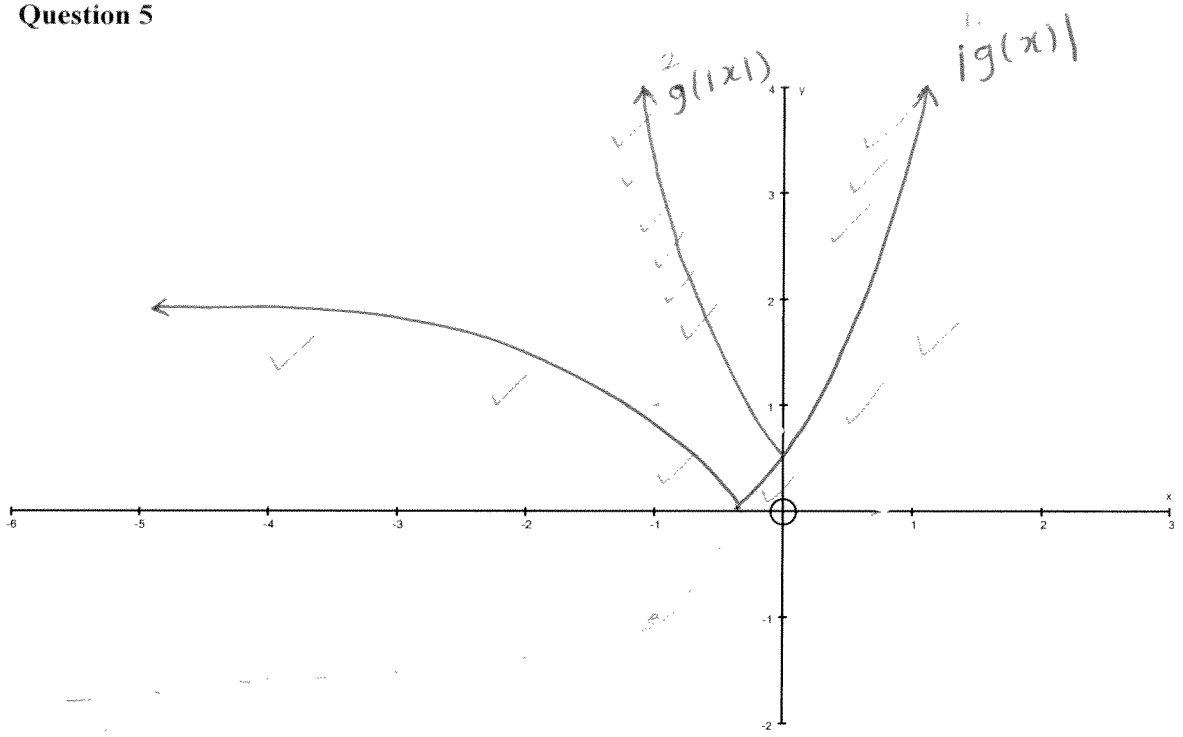
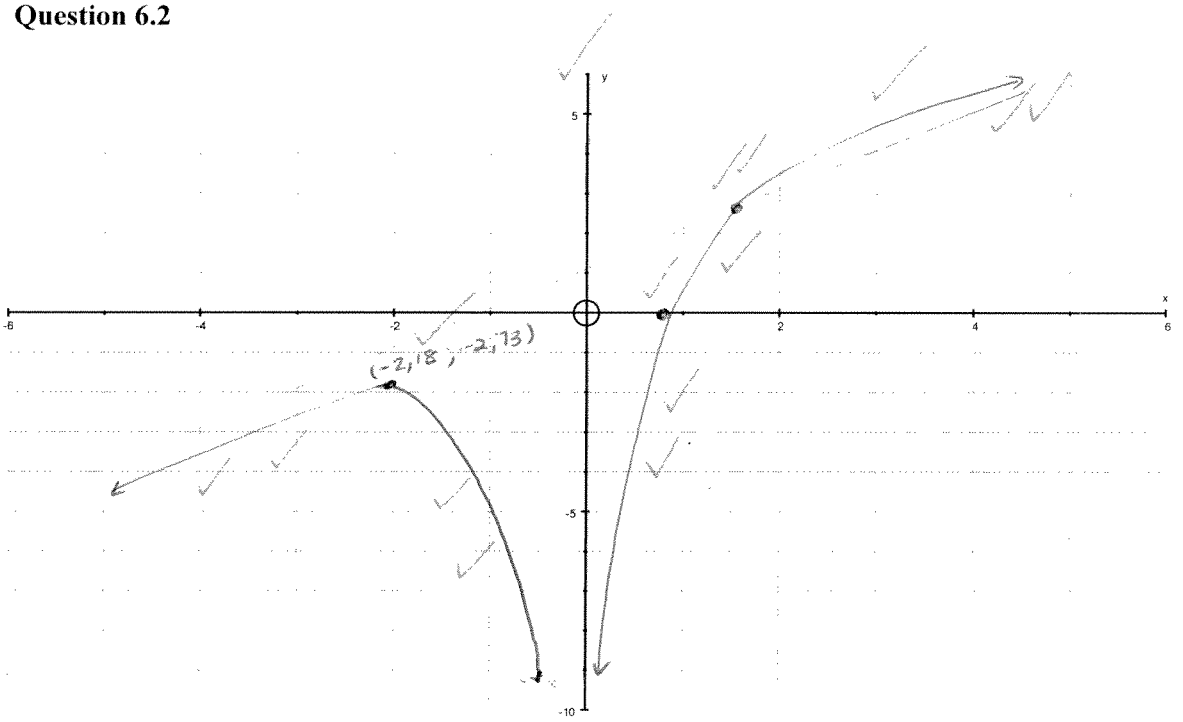
ANSWER SHEET

Section A

Question 5



Question 6.2



6.2 $f(x) = \frac{x^3+x^2+2x-3}{x^2} = x+1+\frac{2x-3}{x^2}$

y int : none

x int : (0,84 ; 0)

Asymptotes: $x = 0$

$y = x+1$ (cut at $x = \frac{3}{2}$)

TP : $f'(x) = 1 + \frac{2(x^2)-(2x-3).2x}{x^4}$

$= \frac{x^4+2x^2-4x^2+6x}{x^4}$

$0 = \frac{x^4-2x^2+6x}{x^4}$

$0 = \frac{x(x^3-2x+6)}{x^{4\ 3}}$

$x = -2,18$

$y = -2,73$

[25]

Question 7

7.1 $l = \theta r$

$\frac{320\pi}{3} = \frac{5\pi}{6}.r$

$128 = r$

$\frac{5}{8}AO = CO$

$CO = 80$ units

6

7.2 $A_{EFOB} = \frac{1}{2}.128^2.\frac{\pi}{6} - \frac{1}{2}.80^2.\frac{\pi}{6}$

$= 832\pi$ units^2

$E\hat{O}B = \frac{\pi}{6}$

7

[13]

Question 8

8.1 $900 = 2(2x \cdot 3x) + 2(2x \cdot h) + 2(3x \cdot h)$

$900 = 12x^2 + 4xh + 6xh$

$900 - 12x^2 = h(4x + 6x)$

$\frac{900 - 12x^2}{10x} = h$

$V = 3x \cdot 2x \cdot h$

$= 3x \cdot 2x \cdot \left(\frac{900 - 12x^2}{10x}\right)$

$= \frac{6x^2(900 - 12x^2)}{10x}$

$= 540x - \frac{36x^3}{5}$ 10

8.2 $\frac{dV}{dx} = 540 - 3\left(\frac{36}{5}\right)x^2$

$0 = 540 - \frac{108}{5}x^2$

$\frac{108x^2}{5} = 540$

$x^2 = 25 \qquad x = 5$ 5

15 by 10 by 12

[15]

Question 9

9.1 $x^3 + y^3 = 6xy$

9.1.1 $3x^2 + 3y^2 \cdot \frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$

$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$ 6

9.1.2 $\frac{dy}{dx}$ at $(3;3) = \frac{6(3) - 3(3)^2}{3(3)^2 - 6(3)} = -1$ 6

$y - 3 = -1(x - 3) \qquad y = -x + 6$

9.2 $y = \frac{1}{x^2}$ $y = x$ $y = 8x$

$\frac{1}{x^2} = x$ $\frac{1}{x^2} = 8x$

$1 = x^3$ $\frac{1}{8} = x^3$

$1 = x$ $\frac{1}{2} = x$

$$A = \int_0^{\frac{1}{2}} 8x - x \, dx + \int_{\frac{1}{2}}^{1} \frac{1}{x^2} - x \, dx$$

$$= \int_0^{\frac{1}{2}} 7x \, dx + \int_{\frac{1}{2}}^{1} \frac{1}{x^2} - x \, dx$$

$$= \frac{3}{2} \text{ units}^2$$

13

[25]

Question 10

10.1 $$V = \pi \int_{\frac{1}{2}}^{2} \left(-x^3 + x^2 - \frac{1}{\sqrt{x}} + 5 \right)^2 dx$$

$$= 51{,}93 \text{ units}^3$$

8

10.2 10.2.1 $$\int \frac{x}{\sqrt{x^2+4}} \, dx$$

$u = x^2 + 4$

$\frac{du}{dx} = 2x$

$\frac{1}{2} du = x \, dx$

$$= \frac{1}{2} \int \frac{1}{\sqrt{u}} \, du$$

$$= \frac{1}{2} \int u^{-\frac{1}{2}} \, du$$

$$= \frac{1}{2} \cdot \frac{u^{\frac{1}{2}}}{1/2} + C$$

$$= \sqrt{x^2+4} + C$$

7

10.2.2 $\int \frac{1-\tan^2\theta}{\sec^2\theta}\,d\theta$

$\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1}$

$= \int \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}\,d\theta$

$= \int \cos^2\theta - \sin^2\theta\,d\theta$

$= \int \cos 2\theta\,d\theta$

$= \frac{\sin 2\theta}{2} + C$

9

10.3 $\int_{3/2}^{3} \frac{\sqrt{9-x^2}}{x^2}\,dx$

$x = 3\sin\theta$

$\frac{dx}{d\theta} = 3\cos\theta$

$dx = 3\cos\theta\,d\theta$

Boundaries

$3 = 3\sin\theta$

$\theta = \frac{\pi}{2}$

$3/2 = 3\sin\theta$

$\frac{\pi}{6} = \theta$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sqrt{9-9\sin^2\theta}}{9\sin^2\theta} \cdot 3\cos\theta\,d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sqrt{9\cos^2\theta}}{9\sin^2\theta} \cdot 3\cos\theta\,d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{3\cos\theta \cdot 3\cos\theta}{9\sin^2\theta}\,d\theta = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{9\cos^2\theta}{9\sin^2\theta}\,d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2\theta\,d\theta = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \operatorname{cosec}^2\theta - 1\,d\theta$

$= -\cot\theta - \theta\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = -\frac{\cos\theta}{\sin\theta} - \theta\Big]_{\pi/6}^{\pi/2}$

$= 0,68 \text{ units}^2$

16

[40]

[200]

Section B

Question 1

1.1 No, reduces by a % of previous area. Will be close to 0, but theoretically never reach 0. ✓✓ 2

1.2 $35(1-0,0325)^{7}$ ✓

$= 27,77\ m^2$ ✓✓✓✓ 4

1.3 $13,89 = 27,77(1-i)^{32}$ ✓ ✓

$\frac{1}{2} = (1-i)^{32}$ ✓

$\sqrt[32]{\frac{1}{2}} = 1-i$ ✓

$0,0214 = i$ ✓

$2,14\% = i$ 5

1.4 Product A ✓ 1

[12]

Question 2

2.1 2.1.1 $800\,000 = \frac{x\left(1-\left(1+\frac{0,12}{12}\right)^{-240}\right)}{\frac{0,12}{12}} + x$ ✓ ✓ ✓ ✓

$x = R\,8712,75$ ✓ 5

2.1.2 $800000\left(1+\frac{0,12}{12}\right)^{60} - \frac{8712,75\left(\left(1+\frac{0,12}{12}\right)^{60}-1\right)}{\frac{0,12}{12}} - \frac{8712,75}{\left(1+\frac{0,12}{12}\right)^{60}}$ ✓ ✓ ✓ ✓

$= R\,725\,961,52$ ✓ 6

2.1.3 $725\,961,52 = \frac{x\left(1-\left(1+\frac{0,1325}{12}\right)^{-180}\right)}{\frac{0,1325}{12}}$ ✓ ✓ ✓ ✓

$x = R\,9304,91$ ✓ ✓ 8

2.2 2.2.1 $718\,330,33 = \dfrac{9000\left(1-\left(1+\frac{0,1325}{12}\right)^{-n}\right)}{\frac{0,1325}{12}}$

$\left(1+\frac{0,1325}{12}\right)^{-n} = 0,1187\ldots$

$\log_{\left(1+\frac{0,1325}{12}\right)} 0,1187\ldots = -n$

$n = 194,06$

$= 195$ months

8

2.2.2 $718\,330,33 = \dfrac{9000\left(1-\left(1+\frac{0,1325}{12}\right)^{-194}\right)}{\frac{0,1325}{12}} + y\left(1+\frac{0,1325}{12}\right)^{-19}$

7

$y = R564,43$

2.3 A : $8712,75 \times 61 + 9304,91 \times 180 = R\,2206361,55$

B : $9000 \times 254 + 564,43 = R\,2286564,43$

4

Should have gone with option A.

[38]

Question 3

$u_6 = x$

$x = 0,8(-1,44) + a(0,7)$

$x = -1,152 + 0,7a$

$-0,4816 = 0,8(x) + a(-1,44)$

$x = -0,602 + 1,8a$

$-1,152 + 0,7a = -0,602 + 1,8a$

$-0,55 = 1,1a$

$-\frac{1}{2} = a$

[8]

Question 4

4.1 $\frac{2}{10} = \frac{1}{5} = 5$ years ✓✓✓ 3

4.2 0,08 ✓ 1

4.3 $f.b = 0,000135$

$f.0,08 = 0,000135$

$f = 0,0016875$ ✓✓ 2

4.4 1500 Rabbits ✓

14 Foxes ✓ 2

4.5 1450 Rabbits ✓

7 Foxes ✓ 2

4.6 $F^* = F^* + 0,000135.R^*F^* - 0,2F^*$ ✓

$0,2F^* = 0,000135 R^*F^*$ ✓✓

$1481,48 = R^*$

Rabbits = $1481,5 \sim 1482$

$R^* = R^* + 0,73R^*\left(1 - \frac{R^*}{5000}\right) - 0,08 R^*F^*$ ✓ ✓

$0 = 0,73(1481,5)\left(1 - \frac{1481,5}{5000}\right) - 0,08(1481,5).F^*$

$F^* = 6,42$ ✓

Foxes = $6,42 \sim 7$ ✓ 9

[19]

Question 5

5.3 $y = 0,3991 - 0,01048\,x.$ ✓✓✓ 5

5.4 $r = 0,3991 \quad -\frac{r}{k} = -0,01048$ ✓✓ ✓

$-\frac{0,3991}{k} = -0,01048$ ✓

$K = 38,09$ ✓ 6

$P_{n+1} = P_n + 0,3991\,P_n\left(1 - \frac{P_n}{38,09}\right)$

5.5

Actual	Model
16,5	17,5
22	21,3
23,2	25,01
28	28,48
31,3	31,35
31,7	33,56

very accurate.

3

[23]

[100]

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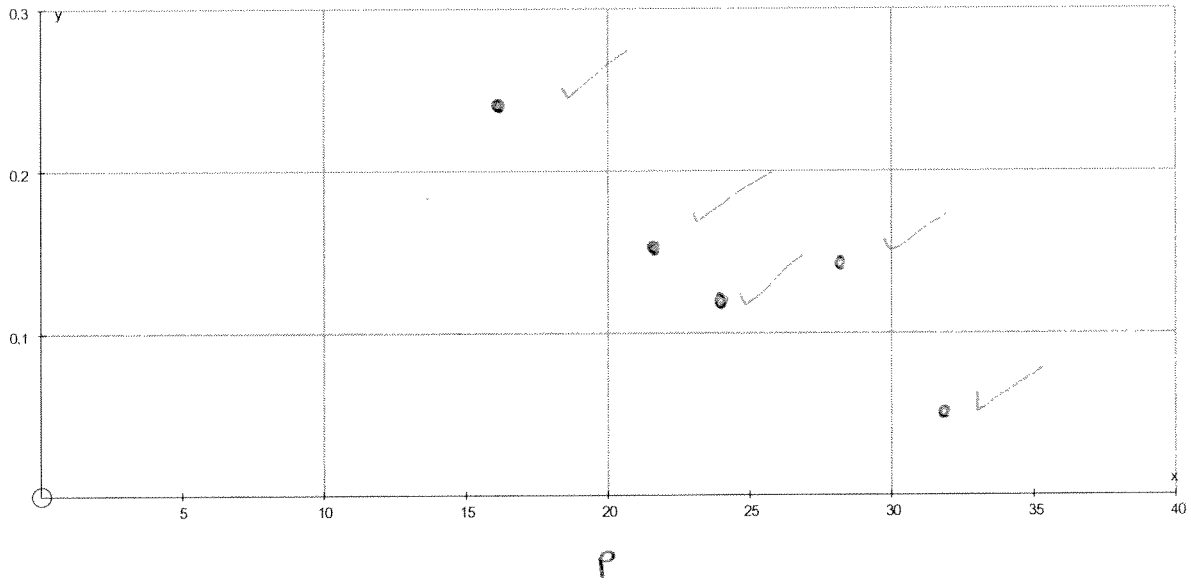
Section B

Question 5.1

Months	**Revenue (millions)**	ΔP	$\frac{\Delta P}{P}$
1	14		
2	16,5	4	0,2424
3	22	3,35	0,1523
4	23,2	3	0,1293
5	28	4,05	0,1446
6	31,3	1,85	0,0591
7	31,7		

4

Question 5.2



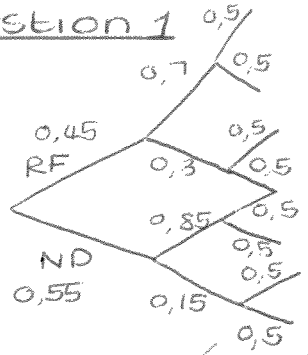
$\frac{\Delta P}{P}$

5

Section C

Question 1

1.1



$P(ND\ wins) = 0,55 \cdot 0,85 + 0,55 \cdot 0,15 \cdot 0,5 + 0,45 \cdot 0,3 \cdot 0,5$

$= 0,57625$ ✓ 4

1.2 $P(RF\ first\ set \mid ND\ wins) = \frac{0,45 \cdot 0,3 \cdot 0,5}{0,57625} = 0,11714$ 7

[11]

Question 2

2.1 $\frac{50!}{15!\,15!\,20!} = 7,31 \times 10^{21}$ $(50C_{20} \cdot 30C_{15} \cdot 15C_{15})$ 4

2.2 Number of ways they can sit together:

$48C_{18} \cdot 30C_{15} \cdot 15C_{15} + 48C_{20} \cdot 28C_{13} \cdot 15C_{15} + 48C_{20} \cdot 28C_{15} \cdot 13C_{13}$

$= 2,387 \times 10^{21}$

Number of ways they can not sit together:

$7,31 \times 10^{21} - 2,387 \times 10^{21}$

$= 4,92 \times 10^{21}$ 8

67.31 %

[12]

Question 3

3.1 3.1.1

$P(X=0) = 8k$

$P(X=1) = 7k$

$P(X=2) = 6k$

$P(X=3) = 5k$

$P(X=4) = 4k$

$P(X=5) = 3k$

$33k = 1$

$k = \frac{1}{33}$ 6

3.1.2 $P[X > 2]$

$P[X = 3] + P[X = 4] + P[X = 5]$

$5\left(\frac{1}{33}\right) + 4\left(\frac{1}{33}\right) + 3\left(\frac{1}{33}\right)$

$= \frac{4}{11} = 0,\dot{3}\dot{6}$

4

[10]

3.2 3.2.1 $\int_0^{1,5} \frac{2}{9} x \, dx = \frac{1}{4}$

7

3.2.2 $\int_0^m \frac{2}{9} x \, dx = \frac{1}{2}$

$\frac{2}{9} \frac{x^2}{2} \Big]_0^m = \frac{1}{2}$

$\frac{m^2}{9} = \frac{1}{2}$

$m^2 = \frac{9}{2}$

$m = \frac{3}{\sqrt{2}}$

7

[14]

Question 4

4.1 $X \sim N(350; 400)$

$P[325 < X < 363]$

$= P[-1,25 < Z < 0,65]$

$z = \frac{x - \mu}{6}$

$z = \frac{325 - 350}{20} = -1,25$

$z = \frac{363 - 350}{20} = 0,65$

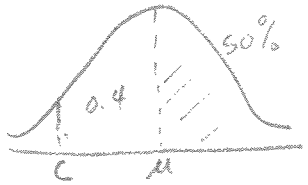
$= P[Z < 0,65] + P[Z < 1,25]$

$= 0,2422 + 0,3944$

$= 0,6366$

8

4.2 $P[X > c] = 0,9$



$0,5 + P[c < X < \mu]$

$0,5 + P\left[0 < Z < \frac{c - 350}{20}\right] = 0,9$

$\frac{c - 350}{20} = 1,29$

$c = 375,8$

$z = \frac{c - 350}{20}$

$P\left[\frac{c-350}{20} < Z < 0\right]$

$P\left[Z < \frac{c-350}{20}\right] = 0,4$

324.20

8

[16]

Question 5

5.1 0,58 ✓✓ There is a moderately strong positive correlation ✓✓ between the goals scored at home and away. 4

5.2 $y = 0,577 + 0,6107x$ ✓✓ ✓✓ 4

5.3 $y = 0,577 + 0,6107(4)$
$= 3,0198$ ✓✓✓ 3

[11]

Question 6

6.1 $\tilde{P} \sim N\left(P ; \frac{P(1-P)}{n}\right)$ $\qquad z = \frac{\hat{P} - P}{\sqrt{\frac{P(1-P)}{n}}}$

$$-2,57 < \frac{\frac{29}{200} - P}{\sqrt{\frac{\frac{29}{200}\left(1-\frac{29}{200}\right)}{200}}} < 2,57$$

$$-0,063986 < \frac{29}{200} - P < 0,063986$$
$$-0,20899 < -P < -0,081$$
$$0,209 > P > 0,081$$
$$0,081 < P < 0,209$$

7

6.2 $$-1,88 < \frac{\hat{P} - P}{\sqrt{\frac{P(1-P)}{n}}} < 1,88$$

$$-1,88\sqrt{\frac{P(1-P)}{n}} < \tilde{P} - P < 1,88\sqrt{\frac{P(1-P)}{n}}$$
$$0,02 - 1,88\sqrt{\frac{P(1-P)}{n}} < \tilde{P} < 0,02 + 1,88\sqrt{\frac{P(1-P)}{n}}$$

$$1,88\sqrt{\frac{P(1-P)}{n}} < 0,04$$
$$\frac{0,02(1-0,02)}{n} < \frac{1}{2209}$$
$$43,3 \leq n$$

Sample of at least 44.

9

[16]

Question 7

Sample mean $\bar{x} = 69,25\% = 0,6925$

Population mean $\mu = 70\% = 0,7$

Standard deviation $\sigma = 0,01$

$H_0 : \mu = 70$

$H_1 : \mu \neq 70$ (Two sided test)

$$z = \frac{0,6925 - 0,7}{\frac{0,01}{\sqrt{87}}} = -2,121$$

Two sided : critical value $= \pm 1,96$

Reject claim.

[10]

[100]